An Optimal Design Technique for Multivariable SISO Controllers

Shuh Woei Yu
Department of Chemical Engineering
National Central University
Chungli, Taiwan, R.O.C.

In the past decades there has been considerable interest in developing control strategies for multivariable processes because of genuine industrial needs. Currently available techniques range from simple combination of feedback loops to full-scale multivariable controllers. However, industrial applications of multivariable control schemes have been rather limited due to design and implementation complexities. Most chemical processes are still being operated with standard PI or PID controllers. Success of this approach in a multivariable environment depends heavily on proper variable pairing and loop tuning. Parameters of the single-input/single-output (SISO) controllers must be specified to provide overall system integrity and satisfactory performance in the face of setpoint change and load disturbance. In addition, optimal controller design is critical not just for pairing the input/output variables but also in analysis and evaluation of the effectiveness of more complex control configurations.

A number of techniques have been proposed to facilitate the design of multivariable feedback controllers. Niederlinski (1971) was the first to recommend the use of maximum gain and critical frequency obtained from continuous cyclic behavior of the system in calculating the settings. The lack of completeness of this method resulted in poor acceptance. The framework of Direct Nyquist Array (DNA) (Rosenbrock, 1974) and related methods such as Inverse Nyquist Array (INA) and Characteristic Loci can also be used in this context. Yet, additional tuning is still required to improve control quality since only the proportional gain based stability region is specified (Waller et al., 1984; Economou and Morari, 1986). Extension of the single-loop Nyquist method to multivariable systems was proposed by Luyben (1986). By setting the biggest log modulus empirically, a detuning factor is calculated and used to relax the diagonal transfer functions-based Ziegler-Nichols settings. Subsequent modifications to the original simple BLT method by adding derivative control action or using a weighted detuning factor for each loop or both were also reported (Monica et al., 1988). Extension of the internal model control (IMC) principles to multiloop systems by treating interactions as perturbations on a SISO structure can also lead to the design of robustly stable controllers (Economou and Morari, 1986; Huang et al., 1987). In this approach, the IMC filters can be used to determine the parameters of the feedback controllers. Yet, none of these methods guarantees system performance since design of the feedback controllers is largely based on the diagonal elements of the transfer function matrix. They are best suited for obtaining a set of initial parameters, and further adjustments have to be carried out to improve control quality. This fine-tuning procedure can be quite tedious when manipulated variable constraints and stability robustness are to be considered. And it may even become impossible as the system dimensions increase.

A different approach to the multiloop SISO controller design problem is the method of inequalities. It was successfully implemented on a pilot packed column and three simulated binary columns by Taiwo (1980). This computer-aided design technique is most noted for its ability in handling manipulated variable constraints directly. Several functionals specifying the desired system behavior such as settling time, overshoot, and the maximal interaction are defined in terms of inequalities. Controller settings that satisfy the inequalities can then be located. Since resetting of the moving boundaries to improve control quality is always required, inconsistency of the solutions may arise (Harris and Mellichamp, 1985).

It is thus obvious that a unified approach to the design of multivariable feedback controllers is still not readily available. Trial-and-error remains an essential part of the design procedures, even though such a process is simplified by the recent development of BLT and IMC methods. Accordingly, the aim of this paper is to propose a design technique for multiloop PI controllers that are capable of providing optimal control quality as well as system integrity.

Algorithm Development

In the absence of a suitable economic measure of control quality in most situations, the second best approach would be to use the system dynamic response in formulating a single performance index whose minimization would provide a set of parameters capable of suppressing undesirable control effects such as overshoot, offset, or oscillation. This is exactly the rationale in devising error-integral tuning formulas for single-feedback loops (Lopez et al., 1967; Rovira, 1981). This study extends this concept to multivariable systems. Integral of the absolute error (IAE) is used as a measure of performance for the purpose of direct comparison with other design methods. Although, in practice, other forms of error integration can be used.

IAE for an $N \times N$ open-loop stable process with N diagonal feedback controllers is simply a summation of the individual loop's absolute error integral.

$$J = \sum_{i=1}^{N} \int |e(t)|\, dt \qquad (1)$$

Since the objective function defined above cannot be easily expressed in terms of tuning constants, numerical methods must be employed to search for their optimal values. In this work, the form of the controllers is taken as PI; therefore, there are a total number of $2N$ constants to be determined.

For most chemical processes, manipulated variables are subject to certain limits to prevent excessive and erratic controller actions. Therefore, in this study, appropriate constraints are assigned to each controller output.

The generalized theorem of Doyle and Stein (1981) for unstructured uncertainties in multivariable systems has been widely accepted as a reliable measure of stability robustness (Luyben, 1986; Economou and Morari, 1986). Hence, the robustness measures of process output and input calculated in the frequency domain are also treated as implicit constraints.

$$DSO = \underline{\sigma}[I + (GB)^{-1}] \qquad (2)$$

$$DSI = \underline{\sigma}[I + (BG)^{-1}] \qquad (3)$$

Yu and Luyben (1986) recommended that all controllers with minimum singular value below 0.3–0.2 be retuned. Hence, lower limits of DSO and DSI are set at 0.3 and 0.2 in this study.

Thus, the design of multivariable feedback controllers becomes a mathematical optimization problem. For an $N \times N$ system with N diagonal PI controllers, there are $2N$ tuning parameters to be determined that will minimize the objective function, defined in Eq. 1, subject to N manipulated variable constraints

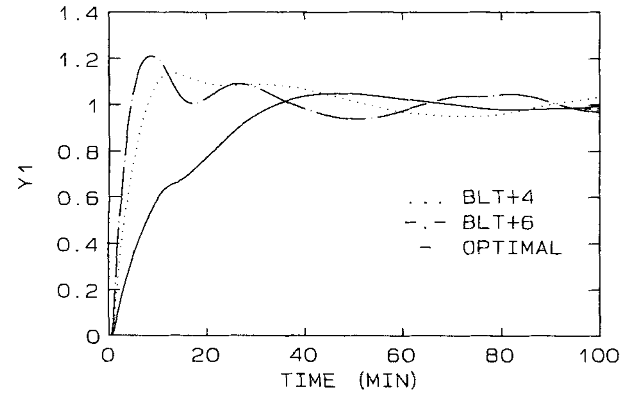

Figure 1. Loop 1 setpoint responses of example 1.

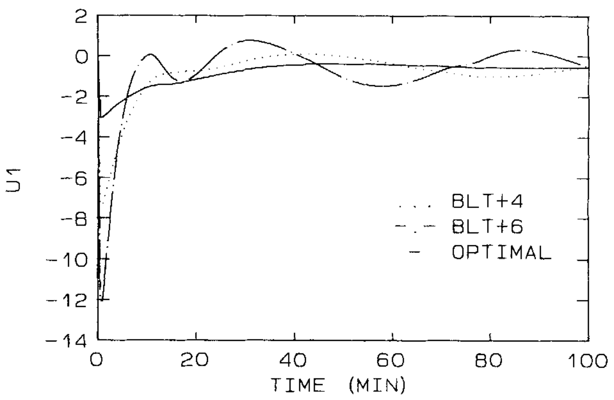

Figure 2. Loop 1 controller actions of example 1.

and two stability robustness requirements defined in Eqs. 2 and 3. The constrained multivariable minimization algorithm of Rosenbrock (Kuester and Mize, 1973) is used in this study. The final tuning is also checked for closed-loop stability by using a multivariable Nyquist plot (Luyben, 1986).

$$W = -1 + det[I + GB] \qquad (4)$$

W is plotted as a function of frequency to check for the encirclement of the $(-1, 0i)$ point.

Examples and Discussion

The proposed design method has been tested on numerous multivariable distillation column examples available in the open literature ranging from 2—to 4—loop systems. For illustration purposes, two complex systems are presented here. Process open-loop transfer functions, labelled T4 and A1 in the original paper, are taken directly from Luyben (1986). The linear, time-invariant transfer functions are transformed into a set of ordinary differential equations and solved with an increment size of 0.1 min. Process dead time is approximated with a delay table. Dynamic responses of the process will provide the accumulated absolute error and manipulated variable adjustment of each loop. Time limit used in the error integration is set at at least 2.5 times the largest lag.

The first test case was chosen for its extreme complexities. For instance, one of the loops has a lag time of 400 min and a process dead time of 60 min. Another distinct feature of the pro-

Table 1. Loop Characteristics and Performance of Example 1

Diagonal RGA	Tuning Method	K_c	T_I
1.09, 0.1, 0.1	BLT+4	−7.04, −2.2, −0.114	11.3, 23,2, 24,2
	BLT+6	−11.26, −3.52, −0.182	7.09, 14.5, 15.1
	optimal	−3.03, −1.59, −0.54	62.0, 88.4, 61.0

		IAE			
DSO	DSI	Loop 1	Loop 2	Loop 3	Total
0.08	0.02	114.	6.4	30.2	48.0
0.07	0.01	10.0	9.3	34.3	53.6
0.48	0.20	13.7	1.8	1.9	17.4

IAE was calculated at the final steady state.

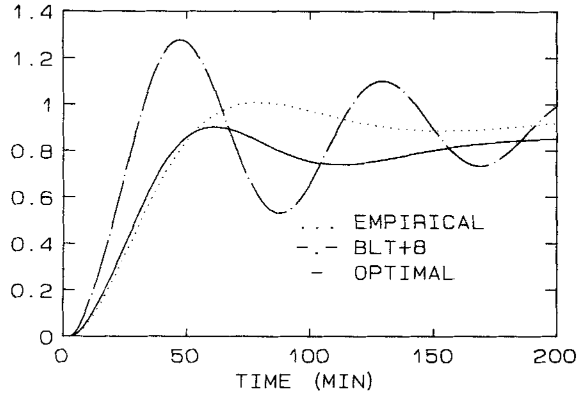

Figure 3. Loop 1 setpoint responses of example 2.

cess is the extremely low diagonal RGA elements of loops 2 and 3.

Note the excellent setpoint tracking ability of the BLT tunings, obtained by setting the biggest log modulus equal to +4-dB (BLT+4) and +6-dB (BLT+6) respectively, for a step change in the loop 1 setpoint as shown in Figure 1. However, this was accomplished with excessive adjustments of the loop 1 manipulated variable, as illustrated by Figure 2. Consequently, the overall system performance was highly unacceptable because of oscillatory behaviors of $Y2$ and $Y3$. Since control performances of all loops were considered, the proposed design technique resulted in excellent control of the whole system. It was brought to the final steady-state conditions in about 80 min. The only drawback might be the slightly sluggish response of the first loop. Table 1 highlights the improvement in performance and stability robustness of the proposed tuning method. All three tuning methods satisfy the stability requirement defined in Eq. 4.

The second case considered was a system with four control loops. Again, a setpoint change in the first loop was introduced. Responses of Y_1 under empirical, BLT+8, and optimal tuning are given in Figure 3. Performance of the individual loops listed in Table 2 indicates that the major shortcoming of the trial-and-error tuning is the severe imbalance among the loops, particularly loops 3 and 4. This tuning was essentially conducted on the premise that the loop with the setpoint change was given the most consideration or weight. Thus, sacrifice in control quality of other loops was inevitable. The BLT+8 tuning method resulted in sustained oscillation in all loops because of the small integral time constants. The optimal tuning was able to bring the whole system to the desired operating conditions with excellent speed and limited amount of overshoot. It was observed that smooth adjustments of each process input were maintained by the empirical and optimization tuning methods while the BLT+8 design caused excessive changes of all four manipulated variables. Table 2 also compares robustness measures of the tuning methods in this test. The extremely low DSO of BLT+8 design might lead to stability problems when changes in process parameters occur. A check of the multivariable Nyquist plot shows that the designed PI controllers all comply with the stability requirement.

Effects of modeling error on robustness properties of the different designs of the test examples were analyzed via a series of simulation experiments. Process gains of the exact model were perturbed simultaneously according to the value of a scaling factor, α. And a second parameter, β, was chosen to determine the ratio of the time constants in the incorrect model to those of the exact model. Thus, different degrees of mismatch can be simulated by varying α and β. Tuning constants determined with the design methods cited above for the exact models were then applied to control the mismatched models for the same step change in the loop 1 setpoint. The results are summarized in Table 3. Variations of IAE indicate sensitivity of the controllers to modelling errors. Thus, it can be concluded from the test results that the proposed multiloop feedback design technique is capable of providing excellent control quality and robustness properties.

Table 2. Loop Characteristics and Performance of Example 2

Diagonal RGA	Tuning Method	K_c	T_I
3.53, 2.77	empirical	0.9, 1.2, 1.0, 0.8	30, 20, 23, 45
2.26, 1.48	BLT+8	2.82, 2.94, 1.18, 2.02	72.2, 7.48, 7.39, 27.8
	optimal	1.14, 1.83, 3.71, 1.67	63.2, 21.8, 8.8, 42.3

		IAE				
DSO	DSI	Loop 1	Loop 2	Loop 3	Loop 4	Total
0.26	0.32	53.1	7.6	67.9	131.9	260.5
0.07	0.11	65.7	5.5	101.7	93.2	266.1
0.30	0.22	88.0	5.6	9.2	59.4	162.2

IAE was calculated at the final steady state.

Table 3. Sensitivity Analysis of the Test Examples

			IAE		
No.	α	β	BLT+4	BLT+6	Optimal
1	1.25	1.0	39.4	28.5	15.0
	1.0	1.25	81.6	231.4	21.5
	1.0	1.0	48.0	53.6	17.4
	1.2	0.9	32.4	27.5	14.2
	1.1	0.8	27.2	28.4	14.1
	0.8	0.8	33.0	45.5	17.8
			Empirical	BLT+8	Optimal
2	1.25	1.0	217.0	623.6	136.2
	1.0	1.25	271.4	243.6	164.1
	1.0	1.0	260.5	266.1	162.2
	1.2	0.9	219.4	858.0	138.9
	1.1	0.8	236.1	762.7	148.3
	0.8	0.8	323.7	204.3	199.7

Notation

B = feedback controller transfer function matrix
e = feedback error
G = process transfer function matrix
I = identity matrix
J = performance index
K_c = proportional gain
N = system order
t = time
T_I = integral time constant, min
u = manipulated variable vector
$W = -1 + \det(I + GB)$
Y = controlled variable vector
$\underline{\sigma}$ = minimum singular value

Literature Cited

Doyle, J. C., and G. Stein, "Multivariable Feedback Design: Concepts for a Classical/Modern Synthesis," *IEEE Trans. Autom. Control,* AC**26**(1), 4 (1981).

Economou, C. G., and M. Morari, "Internal Model Control: VI. Multiloop Design," *Ind. Eng. Chem. Proc. Des. Dev.,* **25**(2), 411 (1986).

Harris, S. L., and D. A. Mellichamp, "Controller Tuning Using Optimization to Meet Multiple Closed-Loop Criteria," *AIChE J.,* **31**(3), 484 (1985).

Huang, H. P., H. S. Lin, and Y. C. Chao, "Multiloop SISO Controller Design," *J. of Chinese Inst. of Chem. Eng.,* **18**(2), 117 (1987).

Kuester, J. L., and J. H. Mize, "Optimization Techniques with Fortran," McGraw-Hill, Inc., New York, 1973.

Lopez, A. M., P. W. Murrill, and C. L. Smith, "Controller Tuning Relationships Based on Integral Performance Criteria," Instrumentation Technology, **14**(11), 57 (1967).

Luyben, W. L., "Simple Method for Tuning SISO Controllers in Multivariable Systems," *Ind. Eng. Chem. Proc. Des. Dev.,* **25**(3), 654 (1986).

Monica, T. J., C. C. Yu, and W. L. Luyben, "Improved Multiloop Single-Input/Single-Output (SISO) Controllers for Multivariable Processes," *Ind. Eng. Chem. Proc. Des. Dev.,* **27**(6), 969 (1988).

Niederlinski, A., "A Heuristic Approach to the Design of Linear Multivariable Interacting Control Systems," *Automatica,* **7**, 691 (1971).

Rosenbrock, H. H., "Computer Aided Control System Design," Academic Press, New York (1974).

Rovira, A. A., PhD dissertation, Department of Chemical Engineering, Louisiana State University, Baton Rouge (1981).

Taiwo, O., "Application of the Method of Inequalities to the Multivariable Control of Binary Distillation Columns," *Chem. Eng. Sci.,* **35**, 847 (1980).

Waller, K. V., K. E. Wikman, and S. E. Gustafsson, "Decoupler Design and Control System Tuning By INA for Distillation Composition Control," Abo Akademi, Finland, Report 84-1 (1984).

Yu, C. C., and W. L. Luyben, "Design of Multiloop SISO Controllers in Multivariable Processes," *Ind. Eng. Chem. Proc. Des. Dev.,* **25**(2), 498 (1986).

Manuscript received Apr. 18, 1989, and revision received Aug. 7, 1989.